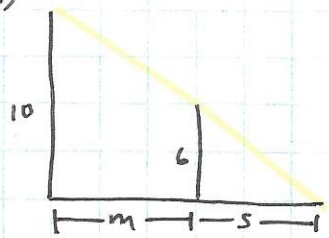
Related Rate 3

1)



$$\frac{10}{m+s} = \frac{6}{s}$$

$$10s = 6m + 6s$$

$$4s = 6m$$

MOMENT	RATE
$m = 10$	$\frac{dm}{dt} = 4$

$$4\frac{ds}{dt} = 6\frac{dm}{dt}$$

$$\frac{ds}{dt} = 6 \text{ ft/sec}$$

$$\frac{ds}{dt} + \frac{dm}{dt} = 10 \text{ ft/sec}$$

The tip of the shadow is moving away from the light post at a rate of 10 ft/sec.

2) $5x^3 + 6y^3 = xy$

MOMENT	RATES
$x = \frac{1}{11}$	$\frac{dy}{dt} = 5$
$y = \frac{1}{11}$	

$$15x^2\frac{dx}{dt} + 18y^2\frac{dy}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$15\left(\frac{1}{121}\right)\frac{dx}{dt} + 18\left(\frac{1}{121}\right)\cdot 5 = \frac{5}{11} + \frac{1}{11}\cdot\frac{dx}{dt}$$

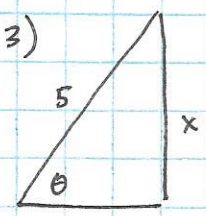
$$\frac{15}{121}\frac{dx}{dt} - \frac{11}{121}\frac{dx}{dt} = \frac{55}{121} - \frac{90}{121}$$

$$\frac{4}{121}\frac{dx}{dt} = -\frac{35}{121}$$

$$\frac{dx}{dt} = -\frac{35}{4} \text{ units/second}$$

The x-coordinate of the point is changing at a rate of $-\frac{35}{4}$ units/second. The point is moving to the left.

3)



MOMENT

$x = 4$

RATES

$\frac{d\theta}{dt} = 2$ rad/sec

$\sin\theta = \frac{1}{5}x$

$\theta = \arcsin\left(\frac{1}{5}x\right)$

$$\frac{d\theta}{dt} = \frac{\frac{1}{5}\frac{dx}{dt}}{\sqrt{1-\left(\frac{1}{5}x\right)^2}}$$

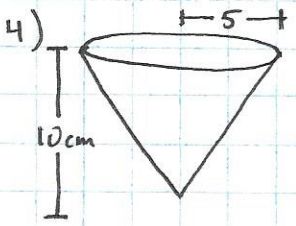
$$2 = \frac{\frac{1}{5}\frac{dx}{dt}}{\sqrt{1-\frac{16}{25}}}$$

$$\frac{2\sqrt{\frac{9}{25}}}{\frac{1}{5}} = \frac{dx}{dt}$$

$\frac{dx}{dt} = 6$ feet/hr

The value of x is increasing at 6 feet/hr.

4)



MOMENT

$h = 5$

$r = \frac{5}{2}$

RATES

$\frac{dh}{dt} = -\frac{3}{10}$ cm/hr

$\frac{dV}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h \rightarrow V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{12}\pi h^3$

$r = \frac{1}{2}h$

a) $h = 5 \quad r = \frac{5}{2}$

$V = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2(5)$

$V = \frac{125\pi}{12}$ cm^3

b) $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

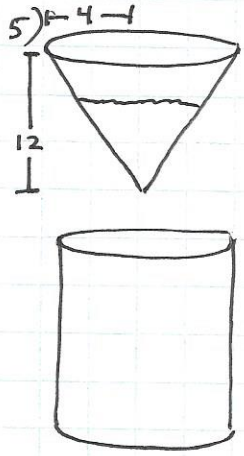
$\frac{dV}{dt} = \frac{1}{4}\pi(5)^2\left(-\frac{3}{10}\right)$

$\frac{dV}{dt} = -\frac{75\pi}{40} = -\frac{15\pi}{8}$ cm^3/hr

c) $\frac{dV}{dt} = k \cdot \pi r^2$

$\frac{1}{4}\pi h^2 \frac{dh}{dt} = k \cdot \pi\left(\frac{1}{2}h\right)^2$

$k = -\frac{3}{10}$

5) 

a) $V = \frac{1}{3}\pi r^2 h$

$r = \frac{1}{3}h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

$V = \frac{1}{27}\pi h^3$

b) $\frac{dh}{dt} = h - 12$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

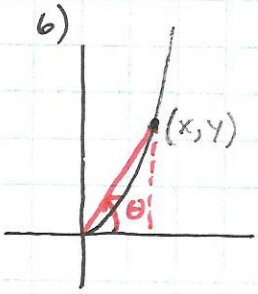
$\frac{dV}{dt} = \frac{1}{9}\pi (3)^2 (3-12) = -9\pi \ ft^3/min$

c) $V = 400\pi y$

$\frac{dV}{dt} = 400\pi \frac{dy}{dt}$

$9\pi = 400\pi \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{9}{400} \ ft/min$

6) 

$y = x^2$ $\quad \tan\theta = \frac{y}{x}$

$\theta = \arctan\left(\frac{y}{x}\right)$

$\theta = \arctan x$

Moment	Rates
$x = 3$	$\frac{dx}{dt} = 10 \ m/sec$

$\frac{d\theta}{dt} = \frac{dx/dt}{1 + x^2}$

$\frac{d\theta}{dt} = \frac{10}{10} = 1 \ rad/sec$